Ch 9 - 474-475: 9, 13, 15, 37, 56

Ch 11 - 567-568: 21, 35-37, 39, 43, 48, 52, 53

#9 $\lim_{t\to 0} \frac{t-\ln(1+2t)}{t^2} \left(\frac{0}{0}\right) = \lim_{t\to 0} \frac{1-\frac{2}{1+2t}}{t^2} \left(\frac{1}{0}\right)$ DNE

#13 $\lim_{x\to\infty} x^{1/x} \rightarrow \frac{1}{x}\ln x = \frac{\ln x}{x} \rightarrow \lim_{x\to\infty} \frac{\ln x}{x} \left(\frac{\infty}{\infty}\right)$

$\Rightarrow \lim_{x\to\infty} \frac{\frac{1}{x}}{1} = \lim_{x\to\infty} \frac{1}{x} = 0 \quad \therefore \lim_{x\to\infty} x^{1/x} = e^0 = 1$

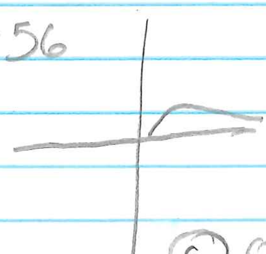
#15 $\lim_{r\to\infty} \frac{\cos r}{\ln r} = 0$ Since $-1 \le \cos r \le 1$ and $\ln r \to \infty$ as $r \to \infty$.

☆Note this is <u>not</u> a L'Hospital situation

#37 $\int_1^\infty \frac{dx}{x^{3/2}} = \lim_{a\to\infty} \int_1^a x^{-3/2}\,dx = \lim_{a\to\infty} -2x^{-1/2}\Big|_1^a$

$= -2 \lim_{x\to\infty} \left(\frac{1}{\sqrt{a}} - 1\right) = -2(0-1) = 2$

#56



(a) $\int_0^\infty x e^{-x/2}\,dx$ (b) $\lim_{a\to\infty} \int_0^a x e^{-x/2}\,dx$

$u = x \quad dv = e^{-x/2}$

$du = dx \quad v = -2e^{-x/2}$

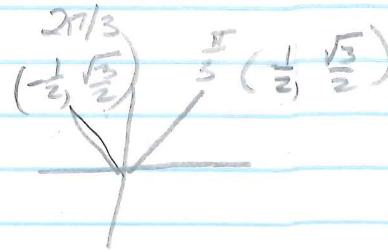
(c) On next page

(c) $\lim_{a\to\infty}\left(-2xe^{-x/2} - (-2)\int_0^a e^{-x/2}\,dx\right)$

$\lim_{a\to\infty}\left(\left(-\frac{2x}{e^{x/2}} + 2(-2)e^{-x/2}\right)\Big|_0^a\right)$

$\lim_{a\to\infty}\left[-\frac{2a}{e^{a/2}} - \frac{4}{e^{a/2}} - (0-4)\right] = 4$

$\boxed{\text{Ch 11}}$



#21 $r = \cos 2\theta \quad x = \cos 2\theta \cos\theta$

$y = \cos 2\theta \sin\theta$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-2\sin 2\theta \sin\theta + \cos 2\theta \cos\theta}{-2\sin 2\theta \cos\theta - \cos 2\theta \sin\theta}\Bigg|_{\theta=\frac{\pi}{3}}$$

$$= \frac{-2\sin\frac{2\pi}{3}\sin\frac{\pi}{3} + \cos 2\pi/3 \cos \pi/3}{-2\sin\frac{2\pi}{3}\cos\frac{\pi}{3} - \cos\frac{2\pi}{3}\sin\frac{\pi}{3}}$$

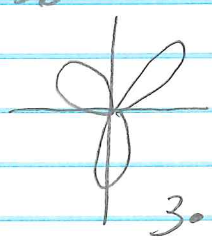
$$= \frac{-\cancel{2}\cdot\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{3}}{\cancel{2}} + \frac{-1}{2}\cdot\frac{1}{2}}{-\cancel{2}\cdot\frac{\sqrt{3}}{\cancel{2}}\cdot\frac{1}{2} - \left(\frac{-1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)} = \frac{-\frac{3}{2} - \frac{1}{4}}{-\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{4}} = \frac{-7}{-\sqrt{3}}$$

$$= \frac{7}{\sqrt{3}}$$

≈ 4.041

#35 $\frac{1}{2}\int_0^{2\pi} (2-\cos 2\theta)^2 \, d\theta \approx 14.137$

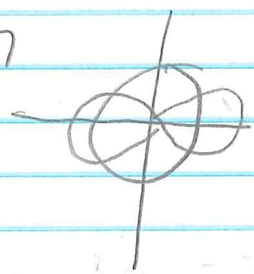
#36



$\sin 3\theta = 0 \to 3\theta = 0, \pi \to \theta = 0, \frac{\pi}{3}$

$$3 \cdot \frac{1}{2} \int_0^{\pi/3} (\sin 3\theta)^2 \, d\theta \approx 0.262$$

#37



$1 + \cos 2\theta = 1$

$\cos 2\theta = 0$

$2\theta = \frac{\pi}{2} \to \theta = \pi/4$

$$A = 2 \cdot \frac{1}{2} \int_{-\frac{\pi}{4}}^{\pi/4} \left[(1+\cos 2\theta)^2 - 1 \right] d\theta$$

$$\approx 2.785$$

#39 $\vec{r}(t) = \langle 4\cos t, \sqrt{2}\sin t \rangle$, $t = \pi/4$

a) $\vec{v}(t) = \langle -4\sin t, \sqrt{2}\cos t \rangle$ $\quad \vec{a}(t) = \langle -4\cos t, -\sqrt{2}\sin t \rangle$

$\vec{v}\left(\frac{\pi}{4}\right) = \left\langle -4 \cdot \frac{\sqrt{2}}{2}, \sqrt{2} \cdot \frac{\sqrt{2}}{2} \right\rangle = \langle -2\sqrt{2}, 1 \rangle$

b) speed $= \sqrt{(-2\sqrt{2})^2 + 1^2} = \sqrt{9} = 3$

#43 $r(t) = \langle -\sin t, \cos t \rangle \quad \vec{r}(0) = \langle 0, 1 \rangle$

$\vec{r}(t) = \langle -\int \sin t\,dt, \int \cos t\,dt \rangle$

$= \langle \cos t + C, \sin t + D \rangle$

Using $\langle 0, 1 \rangle$

$\langle \cos 0 + C, \sin(0) + D \rangle = \langle 0, 1 \rangle$

$1 + C = 0 \qquad D = 1$

$C = -1$

so

$\vec{r}(t) = \langle \cos t - 1, \sin t + 1 \rangle$

#48 $0 \le t \le 4 \qquad x = e^t \cos t \quad y = e^t \sin t$

ⓐ $\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = \left.\dfrac{e^t \cos t + e^t \sin t}{-e^t \sin t + e^t \cos t}\right|_{t=\pi} = \dfrac{e^{\pi}(-1) + e^{\pi} \cdot 0}{-e^{\pi} \cdot 0 + e^{\pi}(-1)}$

$= 1$

ⓑ Speed $= \left.\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\right|_{t=3}$

$= \sqrt{(e^3 \cos 3 + e^3 \sin 3)^2 + (-e^3 \sin 3 + e^3 \cos 3)^2}$

≈ 28.405

#53 $r = \frac{4}{1+\sin\theta}$ $0 \le \theta \le \pi$

ⓐ $A = \int_0^{\pi} \frac{1}{2} r^2 \, d\theta = 10.667$

ⓑ $r = \frac{4}{1+\sin\theta}$

$r(1+\sin\theta) = 4$

$r + r\sin\theta = 4$

$r = 4 - r\sin\theta$

$r = 4 - y$

$r^2 = (4-y)^2$

$x^2 + \cancel{y^2} = 16 - 8y + \cancel{y^2}$

$8y = 16 - x^2$

ⓒ $y = 2 - \frac{x^2}{8}$

Area $= \int_{-4}^{4} \left(2 - \frac{x^2}{8}\right) dx = 10.667$

#52 Part A: $x = t-2, \quad y = (t-2)^2$

Part B: $x = \frac{3}{2}t - 4 \quad y = \frac{3}{2}t - 2$

(a) Part A velocity: $\langle 1, 2(t-2) \rangle \big|_{t=3} = \langle 1, 2 \rangle$

Part B velocity: $\langle \frac{3}{2}, \frac{3}{2} \rangle \big|_{t=3} = \langle \frac{3}{2}, \frac{3}{2} \rangle$

(b) $\int_0^3 \sqrt{1^2 + [2(t-2)]^2}\, dt = 6.126$

(c) Particles collide when their position vectors are the same:

$$\langle t-2, (t-2)^2 \rangle = \langle \tfrac{3}{2}t - 4, \tfrac{3}{2}t - 2 \rangle$$

$t - 2 = \frac{3}{2}t - 4$

$2 = \frac{1}{2}t$

$t = 4$

Verify y-positions are the same at $t = 4$

$(4-2)^2 = \frac{3}{2}(4) - 2$

$4 = 4$ ✓

$\therefore$ particles collide @ $t = 4$.