1-21 odd, 22-24, 27, 31-32, 38, 43, 49, 57, 63
67-69

Ch7 Review

$\frac{\sin\frac{\pi}{3}}{\cos\frac{\pi}{3}} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$

#1 $\int_0^{\pi/3} \sec^2\theta\, d\theta = \tan\theta \Big|_0^{\frac{\pi}{3}} = \tan\frac{\pi}{3} - \tan 0$
$= \boxed{\sqrt{3}}$ ✓

#3 $\int_0^1 \frac{36\,dx}{(2x+1)^3}$

$u = 2x+1$
$du = 2dx$
$\frac{du}{2} = dx$

$\Rightarrow \frac{1}{2}\int \frac{36}{u^3}\,du$

$= 18\int u^{-3}\,du$

$= \frac{18}{-2}u^{-2} = \frac{9}{u^2}$

$= -\frac{9}{(2x+1)^2}\Big|_0^1$

$-1 + 9 = \boxed{8}$ ✓

#5 $\int_0^{\pi/2} 5\sin^{3/2}x\cos x\,dx$

$u = \sin x$
$du = \cos x\,dx$

$\Rightarrow \begin{matrix} u(\frac{\pi}{2}) = 1 \\ u(0) = 0 \end{matrix} \Rightarrow 5\int_0^1 u^{3/2}\,du = 5\cdot\frac{2u^{5/2}}{5}\Big|_0^1 = 5\left(\frac{2}{5} - 0\right)$
$= \boxed{2}$ ✓

#7 $\int_0^{\pi/4} e^{\tan x}\sec^2 x\,dx$

$u = \tan x$
$du = \sec^2 x\,dx$

$u(\frac{\pi}{4}) = \tan\frac{\pi}{4} = 1$
$u(0) = 0$

$\Rightarrow \int_0^1 e^u\,du = \boxed{e - 1}$ ✓

1-21 odd
22-24
27, 31-32, 38, 43, 49, 57, 63, 67-69

#9 $\int_0^1 \frac{x}{x^2+5x+6}\,dx$ * Use partial fractions

$\Rightarrow \frac{x}{(x+3)(x+2)} = \frac{A}{x+3} + \frac{B}{x+2}$

$\Rightarrow x = A(x+2) + B(x+3)$

$x=-2 \quad B=-2$

$x=-3 \quad A=3$

$\Rightarrow \int_0^1 \left(\frac{3}{x+3} - \frac{2}{x+2}\right)dx = 3\ln|x+3| - 2\ln|x+2|$

$= \ln\left|\frac{(x+3)^3}{(x+2)^2}\right| \Big|_0^1 = \boxed{\ln\left(\frac{64}{9}\right) - \ln\left(\frac{27}{4}\right) \text{ or } \approx 0.5212}$ ✓

#11 $\int \frac{\cos x}{2-\sin x}\,dx$ $\quad u = 2-\sin x$

$du = -\cos x\,dx$

$-du = \cos x\,dx$

$\Rightarrow -\int \frac{1}{u}\,du = -\ln|2-\sin x| + C$ ✓

#13 $\int \frac{t\,dt}{t^2+5}$ $\quad u = t^2+5$

$du = 2t\,dt$

$\frac{du}{2} = t\,dt$

$= \frac{1}{2}\int \frac{1}{u}\,du$

$= \frac{1}{2}\ln|t^2+5| + C$ ✓

#15 $\int \frac{\tan(\ln y)}{y}\,dy \qquad u = \ln y \quad du = \frac{1}{y}\,dy$

$= \int \tan u\,du = -\ln\left|\cos(\ln y)\right| + C$

#17 $\int \frac{dx}{x\ln x} \qquad u = \ln x \quad du = \frac{1}{x}\,dx$

$\int \frac{1}{u}\,du = \ln|\ln x| + C$

#19 $\int x^3\cos x\,dx$

u	dv
x^3	$+\ \cos x$
$3x^2$	$-\ \sin x$
$6x$	$+\ -\cos x$
6	$-\ -\sin x$
0	$\cos x$

$= x^3\sin x + 3x^2\cos x - 6x\sin x - 6\cos x + C$

#21 $\int e^{3x}\sin x\,dx \qquad u = e^{3x} \quad dv = \sin x \qquad du = 3e^{3x}\,dx \quad v = -\cos x$

$= -e^{3x}\cos x + 3\int \cos x \cdot e^{3x}\,dx \qquad u = e^{3x} \quad du = 3e^{3x} \quad dv = \cos x \quad v = \sin x$

$= -e^{3x}\cos x + 3\left(e^{3x}\sin x - \int 3e^{3x}\sin x\,dx\right)$

$= -e^{3x}\cos x + 3e^{3x}\sin x - 9\int e^{3x}\sin x\,dx$

so

$10\int e^{3x}\sin x\,dx = -e^{3x}\cos x + 3e^{3x}\sin x$ (see next page)

$$\text{So } \int e^{3x}\sin x\,dx = \frac{3e^{3x}\sin x}{10} - \frac{e^{3x}\cos x}{10} + C \checkmark$$

#22

$$\int x^2 e^{-3x}\,dx = -\frac{1}{3}x^2e^{-3x} - \frac{2xe^{-3x}}{9} - \frac{2}{27}e^{-3x} + C \checkmark$$

u		dv
x^2	+	e^{-3x}
$2x$	−	$-\frac{1}{3}e^{-3x}$
2	+	$\frac{1}{9}e^{-3x}$
0		$-\frac{1}{27}e^{-3x}$

#23

$$\int \frac{25}{x^2-25}\,dx \Rightarrow \frac{25}{(x+5)(x-5)} = \frac{A}{(x+5)} + \frac{B}{(x-5)}$$

$$25 = A(x-5) + B(x+5) \qquad x=5,\ B=\frac{5}{2}$$

$$x=-5,\ A=-\frac{5}{2}$$

$$\int \left(\frac{-5}{2(x+5)} + \frac{5}{2(x-5)}\right)dx$$

$$= \frac{-5}{2}\ln|x+5| + \frac{5}{2}|x-5| + C$$

$$= \frac{5}{2}\ln\left|\frac{x-5}{x+5}\right| + C \checkmark$$

#24 $\int \frac{5x+2}{2x^2+x-1}\,dx \Rightarrow \frac{5x+2}{(2x-1)(x+1)}$

$\Rightarrow \frac{5x+2}{(2x-1)(x+1)} = \frac{A}{2x-1} + \frac{B}{x+1}$

$\Rightarrow 5x+2 = A(x+1) + B(2x-1)$ $\quad x=-1 \Rightarrow -3=-3B$

$B=1$

$x=\frac{1}{2}, \quad \frac{3}{2}A = \frac{5}{2}+2$

$\frac{3}{2}A = \frac{9}{2} \Rightarrow A=3$

$\Rightarrow \int \left(\frac{3}{2x-1} + \frac{1}{x+1}\right)dx$

$u=2x-1$

$\frac{du}{2}=dx$ $\quad \Rightarrow \quad = \frac{3}{2}\ln|2x-1| + \ln|x+1| + C$

$= \ln|2x-1|^{\frac{3}{2}}|x+1| + C$ ✓

or

No need to get it in this form! $\longrightarrow \frac{1}{2}\ln|(2x-1)^3(x+1)^2| + C$

#27 $\frac{dy}{dt} = \frac{1}{t+4} \qquad y(-3)=2$

$\int dy = \int \frac{1}{t+4}\,dt$

$y = \ln|t+4| + C$

$2 = \ln|1| + C$

$2 = C$

$\Rightarrow y = \ln|t+4| + 2$ ✓

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#31 $\frac{dy}{dx} = y+2 \quad y(0)=2$

$\int \frac{dy}{y+2} = \int dx \qquad \ln|y+2| = x + C$

$\ln 4 = C$

$\Rightarrow e^{\ln|y+2|} = e^{x+\ln 4}$

$\Rightarrow |y+2| = e^{x} e^{\ln 4}$

$\Rightarrow y+2 = \pm e^{\ln 4} e^{x}$

$\Rightarrow y = \pm 4e^{x} - 2$ but

$\boxed{y = 4e^{x} - 2}$ satisfies the initial condition

#32 $\frac{dy}{dx} = (2x+1)(y+1) \quad y(-1)=1$

$\int \frac{dy}{y+1} = \int (2x+1)\,dx$

$\ln|y+1| = x^2 + x + C$

$\ln 2 = 1 + (-1) + C \Rightarrow C = \ln 2$

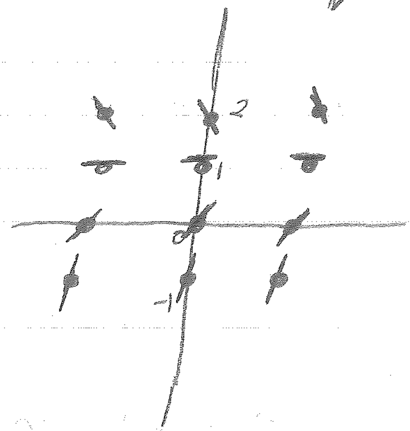
$e^{\ln|y+1|} = e^{x^2+x+\ln 2}$

$|y+1| = e^{x^2+x} e^{\ln 2}$

$\Rightarrow y+1 = \pm 2e^{x^2+x}$ satisfies the initial condition

$\boxed{y = 2e^{x^2+x} - 1}$

#38 $\frac{dy}{dx} = 1-y$



$y=1,\ \frac{dy}{dx}=0$

$y=2,\ \frac{dy}{dx}=-1$

$y=0,\ \frac{dy}{dx}=1$

$y=-1,\ \frac{dy}{dx}=2$

#43 $\frac{dy}{dx} = x+y-1$ $(1,1)$ $\Delta y = \frac{dy}{dx}\Delta x$

start	Δx	$(x+y-1)(0.1) = \Delta y$
$(1,1)$	0.1	0.1
$(1.1, 1.1)$	0.1	$(1.1+1.1-1)(0.1) = \Delta y$ $0.12 = \Delta y$
$(1.2, 1.22)$		$(1.2+1.22-1)(0.1)$ 0.142

$(1.3, 1.362)$ so $y \approx 1.362$ when $x = 1.3$

this is $a(t)$

#49 $\frac{d^2s}{dt^2} = (2+6t)\ m/s^2$ $V(0) = 4\ m/s$

a) $\int (2+6t)\,dx = 2t + \frac{6t^2}{2} = 2t + 3t^2 + C = v(t)$

Since $v(0)=4$, $C=4$ so $v(t) = (2t+3t^2+4)\ m/s$

b) $\int v(t) =$ distance

so

$\int_0^1 2t + 3t^2 + 4\,dt = t^2 + t^3 + 4t \Big|_0^1 = 6\,m$

$6 - 0$

#57 $\frac{dL}{dx} = -kL \qquad \frac{dL}{L} = -k\,dx$

$\Rightarrow e^{\ln L} = e^{-kx + C}$

$\Rightarrow L(x) = e^{C} e^{-kx} \rightarrow e^{C} = L_0$

$\Rightarrow L(x) = L_0 e^{-kx}$

Initial cond: $L(18) = \frac{1}{2}L_0 \Rightarrow \frac{1}{2}\cancel{L_0} = \cancel{L_0} e^{-18k}$

$\frac{1}{2} = e^{-18k}$ store

$\ln(1/2) = -18k$

$k = \frac{\ln(1/2)}{-18} \approx 0.0385..$

Can't work when intensity is $0.1 L_0$

$\Rightarrow 0.1\cancel{L_0} = \cancel{L_0} e^{-kx}$ solve for x

$\frac{\ln(0.1)}{-k} = x \approx 59.795$ ft

stored in calc

#63 Continuous: $A(t) = A_0 e^{rt}$

Finite: $A(t) = A_0 \left(1 + \frac{r}{k}\right)^{kt}$ ($k=1$ since compounded annually)

ⓐ $20{,}000 = 10{,}000 \left(1 + \frac{0.063}{1}\right)^{1 \cdot t}$

$2 = (1 + 0.063)^t$

$\ln 2 = \ln(1.063)^t$

$\ln 2 = t \ln(1.063)$

$t = \frac{\ln 2}{\ln(1.063)} \approx 11.345$ years

#63 (b)

$$20{,}000 = 10{,}000\, e^{0.063t}$$

$$2 = e^{0.063t}$$

$$\ln 2 = 0.063t \Rightarrow t = \frac{\ln 2}{0.063} \approx 11.002 \text{ years}$$ ✓

#67 $\frac{dy}{dt} = 1.2y(1-y)$ * this is logistic!

$$\frac{dy}{dt} = ky(M-y)$$

(a) spreading fastest at 1/2 of carrying capacity so $100\%/2 = \underline{50\% \text{ of townsfolk}}$ ✓

(b) Use logistic solution:

$$y(t) = \frac{M}{1+Ae^{-(Mk)t}}$$

$M=1$

$-(Mk) = -(1 \cdot 1.2) = -1.2$

$$y(t) = \frac{1}{1+Ae^{-1.2t}}$$

* use initial condition to find A

$t=0 \quad y = 10\% \; (0.1)$

$$0.1 = \frac{1}{1+A} \Rightarrow 1+A = 10$$

$$A = 9$$

$$\therefore \; y(t) = \frac{1}{1+9e^{-1.2t}}$$

(c) rumor is spreading fastest at 1/2 of carrying capacity (1/2 of 100%) = 50% = 0.5

(see next page)

(d) $0.5 = \dfrac{1}{1+9e^{-1.2t}}$

$1+9e^{-1.2t} = 2$

$9e^{-1.2t} = 1$

$e^{-1.2t} = \dfrac{1}{9}$

$t = \dfrac{\ln(1/9)}{-1.2} \approx 1.831$ days

directly proportional

#68 $\dfrac{dP}{dt} = k(600-P)$

(a) sep. variables ad solve

$\displaystyle\int \frac{dP}{600-P} = \int k\,dt$

$\ln(600-P) = kt + C$

$P(0)=200$ so $C = \ln 400$

$e^{\ln(600-P)} = e^{kt + \ln 400}$

$600-P = 400e^{kt}$

$P(t) = 600 - 400e^{kt}$

(b) $P(2)=500$ so

$500 = 600 - 400e^{2k}$

$e^{2k} = \dfrac{1}{4} \Rightarrow 2k = \ln(1/4)$

$k = \dfrac{\ln(1/4)}{2}$

$k = -0.693$

(c) $P(t) = 600 - 400e^{-0.693t}$

$\lim_{t\to\infty} e^{-0.693t} = 0 \quad \therefore \quad \lim_{t\to\infty} P(t) = 600$ wolves

#69 $\frac{dv}{dt} = -2(v+17)$ and $v(0) = -47$

(a) $\int \frac{dv}{v+17} = \int -2\,dt$

$\ln|v+17| = -2t + C$

and $v(0) = -47$ so

$\ln|-47+17| = C \Rightarrow C = \ln 30$

$\ln|v+17| = -2t + \ln 30$

$|v+17| = e^{-2t+\ln 30}$

$|v+17| = 30e^{-2t}$

$v = \pm 30e^{-2t} - 17$

* initial condition is satisfied when

$v(t) = (-30e^{-2t} - 17)$ ft/s

(b) terminal vel $= \lim_{t\to\infty} -30e^{-2t} - 17 = -17$ ft/s

since $\lim_{t\to\infty} e^{-2t} = 0$

(c) $v(t) = -30e^{-2t} - 17$

$|v(t)| = 20$ so $v(t) = -20$ ft/s

speed

$$-20 = -30e^{-2t} - 17$$

$$\frac{-3}{-30} = e^{-2t}$$

$$\frac{1}{10} = e^{-2t}$$

$$\approx \frac{\ln(1/10)}{-2} = t \approx 1.151 \text{ sec}$$